COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

HIGH SCHOOL GEOMETRY: COMPANY LOGO

UNIT OVERVIEW

This 3-4 week unit uses an investigation of rigid motions and geometric theorems to teach students how to verify congruence of plane figures and use the implications of congruence to solve problems and create proofs about geometric relationships. Students will demonstrate mastery of the content by making sense of the Company Logo Performance Task and persevering in solving the task.

TASK DETAILS

Task Name: Company Logo

Grade: High School

Subject: Geometry

Depth of Knowledge: 3

Task Description: This task asks students to visualize geometric shapes, identify plane figures and their attributes, prove triangles are congruent, determine the area of quadrilaterals, make geometric conjectures and justify geometric arguments.

Standards Assessed:
G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
***G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
***G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
***G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant.

Standards for Mathematical Practice:
MP. 1 Make sense of problems and persevere in solving them.
MP. 3 Construct viable arguments and critique the reasoning of others.
MP. 6 Attend to precision.
**Depending on the student's solution path they may not demonstrate understanding of these standards. We included standards that we saw demonstrated in the student work samples.*

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

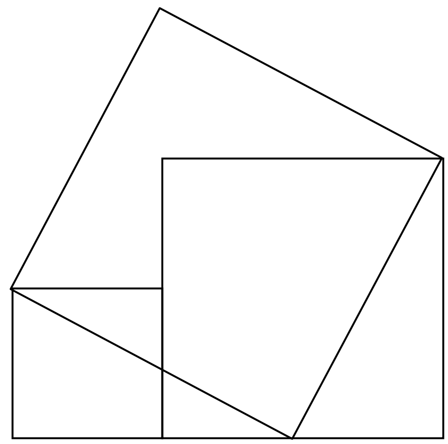
Acknowledgements: The unit outline was developed by Mike Stevens with input from Curriculum Designers Alignment Review Team. The tasks were developed by the schools in the 2010-2011 NYC DOE High School Performance Based Assessment Pilot, in collaboration with the Silicon Valley Mathematics Initiative and SCALE.

HIGH SCHOOL GEOMETRY: COMPANY LOGO PERFORMANCE TASK

Performance Task

The Company Logo

A company has designed a new logo using overlapping squares.

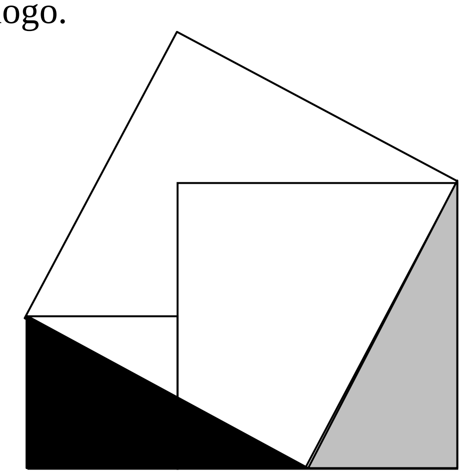


1. How many squares do you see in the logo? ____________________________

   Describe where you see the squares.

2. The logo designer colored two triangles in the logo.

   How are the two triangles related?
   Justify your answer.



3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

HIGH SCHOOL GEOMETRY: COMPANY LOGO

UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

Math Grade H.S. – Company Logo
Common Core Learning Standards/
Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The "what" of learning.* How does the task present information and content in different ways? How students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

In this task, teachers can…

- **Make explicit links between information provided in texts and any accompanying representation of that information in illustrations, equations, charts, or diagrams** by reviewing mathematical definitions and have the students create accompanying examples.

ACTION/EXPRESSION: *The "how" of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can…

- **Use social media and interactive web tools (e.g., discussion forums, chats, web design, annotation tools, storyboards, comic strips, animation presentations)** by asking students to find examples of logos and advertisements, as well as incorporate software to identify, measure, and manipulate geometric shapes.

ENGAGEMENT: *The "why" of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can…

- **Optimize relevance, value and authenticity** by including activities that foster the use of imagination to solve novel and relevant problems, or make sense of complex ideas in creative ways**.**

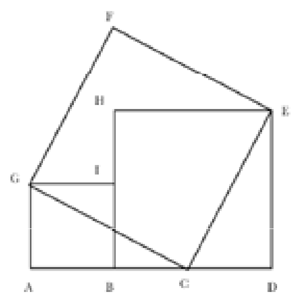
Visit http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm to learn more information about UDL.

HIGH SCHOOL GEOMETRY: COMPANY LOGO RUBRIC

The rubric section contains a scoring guide and performance level descriptions for the Company Logo task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

The Company Logo		Rubric
The elements of performance required by this task are: • Visualizes geometric shapes, identifies plane figures and their attributes, proves triangles are congruent, determines the area relationships of quadrilaterals, makes geometric conjectures and proves/justifies geometric arguments.	Points	Section Points
1. There are three squares. Describes the square such as: The figure contains a small square ◊ABIG that shares an adjacent side with a medium size square ◊BDEH. There is a large square ◊CEFG that intersects the small square at vertex G and the medium square at vertex E and point C. 	1 1	2
2. The two triangles are congruent (ΔACE = ΔDEC). Justifies answer such as: Both triangles ΔACE and ΔDEC are right triangles because they share an angle with a square, <A and <D. Both hypotenuses are congruent because they both share a side of the large triangle, GC = CE. <CED = <GCA because the are both complements of the same angle, <ECD. Therefore the two triangles are congruent by the Hypotenuse Angle Theorem. *Partial Credit* Some correct geometric reasoning	1 3 (1)	4
3. The sum of the areas of the two smaller squares is equal area of the largest square. Justify there conjecture such as: From part two, ΔACE = ΔDEC. Therefore in both triangles the small leg is the length of the small square, the other leg is the length of the medium square, and the hypotenuse is the length of the large square. Using the Pythagorean Theorem the sum of area of the two squares (small and medium) equals the area of the largest square. Accept proof that two outside triangles fit exactly inside large square along with the remaining parts of the smaller square, either through transformations or Euclidean theorems.	1 3	4
Total Points		**10**

High School Geometry: Company Logo
Rubric

Performance Level Descriptions and Cut Scores

Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0-2 points)

The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The student struggles to recognize patterns or the structure of the problem situation.

Level 2: Performance Below Standard (3-4 points)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student's response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (5-6 points)

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could fix with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. Student makes sense of quantities and their relationships in the problem situations. They often use abstractions to represent a problem symbolically or with other mathematical representations. The student may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (7-10 points)

The student's response meets the demands of nearly all of the task, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in construction arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. Student looks closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

HIGH SCHOOL GEOMETRY: COMPANY LOGO
ANNOTATED STUDENT WORK

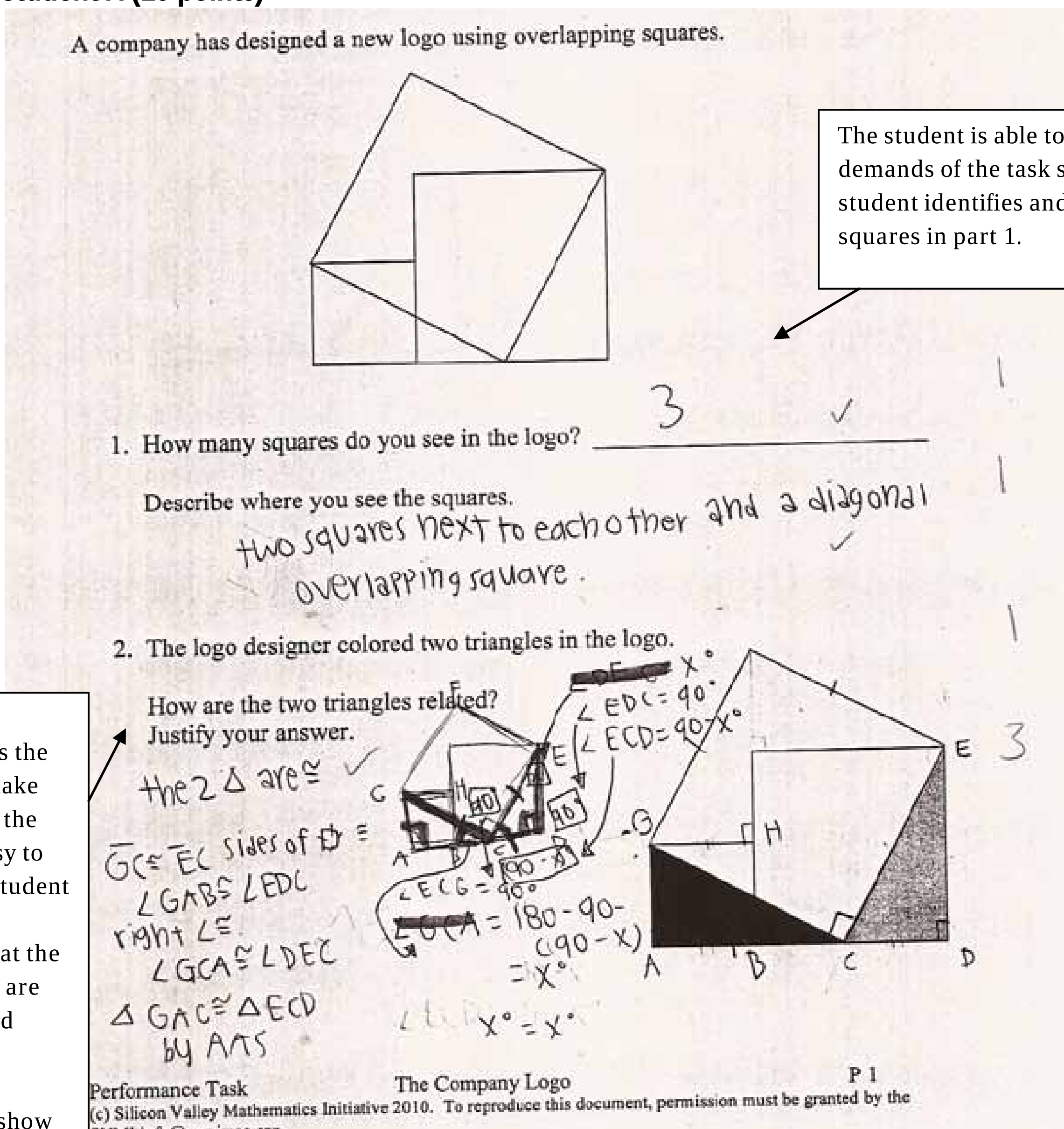
This section contains annotated student work at a range of score points and implications for instruction for each performance level. The student work shows examples of student understandings and misunderstandings of the task, which can be used with the implications for instruction to understand how to move students to the next performance level.

High School Geometry: Company Logo
Annotated Student Work

Level 4: Achieves Standards at a High Level – Score range 7-10

The student's response meets the demands of nearly all of the task, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in construction arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. Student looks closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

Student A (10 points)


A company has designed a new logo using overlapping squares.

1. How many squares do you see in the logo? 3 ✓

Describe where you see the squares.

two squares next to each other and a diagonal overlapping square. ✓

2. The logo designer colored two triangles in the logo.

How are the two triangles related?
Justify your answer.

the 2 Δ are ≅ ✓

$\overline{GC} \cong \overline{EC}$ sides of ⊡ ≅

∠GAB ≅ ∠EDC right ∠ ≅

∠GCA ≅ ∠DEC

Δ GAC ≅ Δ ECD by AAS

[redacted] x°

∠EDC = 90°

∠ECD = 90 − x°

∠ECG = 90°

[redacted] = 180 − 90 − (90 − x)

= x°

x° = x°

3

Performance Task The Company Logo P 1

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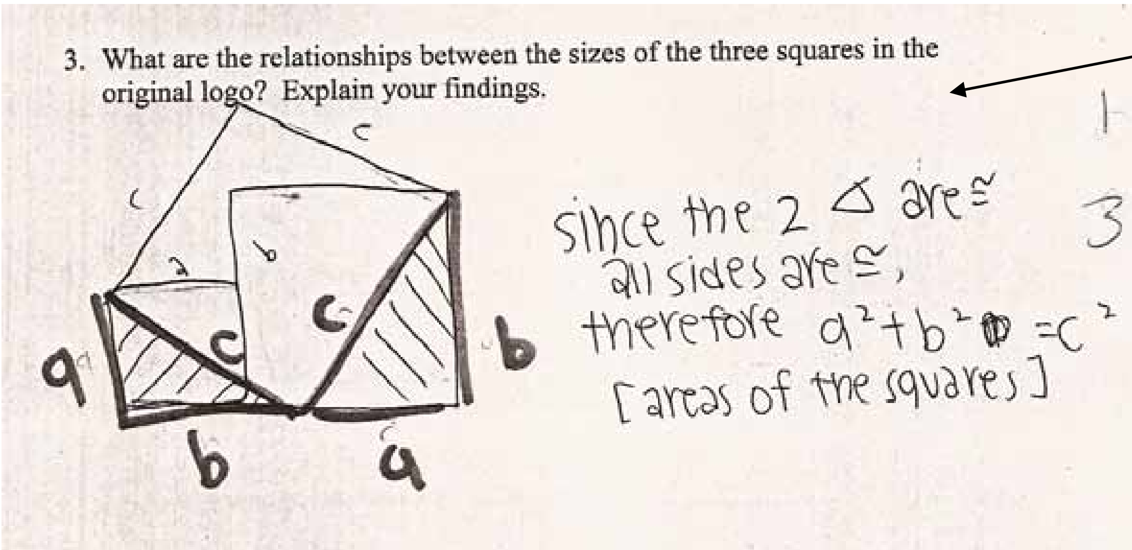
The student is able to meet all the demands of the task satisfactorily. The student identifies and locates the squares in part 1.

In part 2 the student labels the diagram to make discussion of the argument easy to follow. The student makes the conjecture that the two triangles are congruent and develops a convincing argument to show using Angle, angle side theorem to show that this is true (MP #3 – construct arguments, CCSS – congruence).

High School Geometry: Company Logo
Annotated Student Work

Student A (cont'd)



3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

since the 2 $\triangle$ are $\cong$
all sides are $\cong$,
therefore $a^2 + b^2 = c^2$
[areas of the squares]

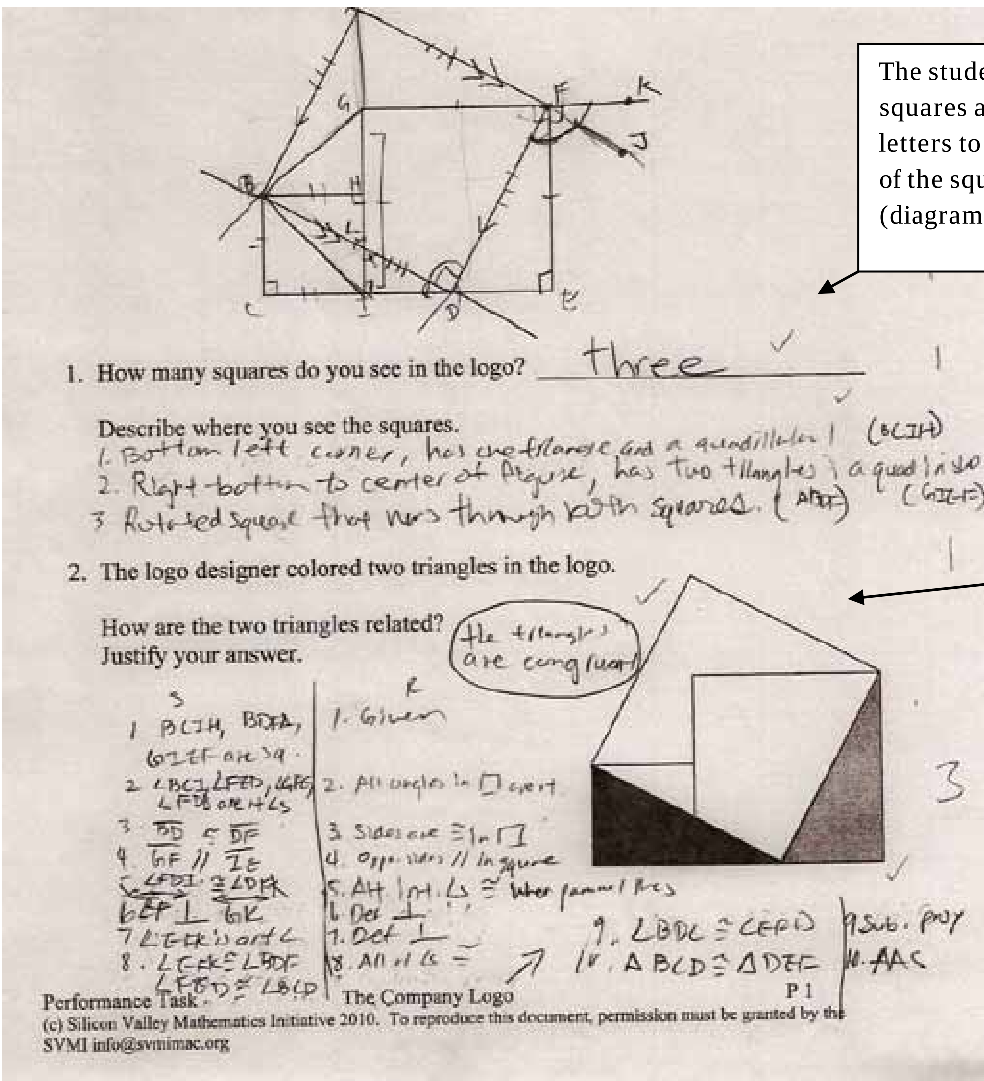
In part 3 the student is able to use area formulas and apply the Pythagorean Theorem to show that the area of the smaller squares is equal to the area of the larger square. (MP#1, CCSS – Congruence Pythagorean Theorem). The student can construct proofs and understands the role of axioms and definitions (MP #3 – construct viable arguments).

High School Geometry: Company Logo
Annotated Student Work

Level 3: Performance at Standard – Score range 5-6

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could fix with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. Student makes sense of quantities and their relationships in the problem situations. They often use abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Student B (6 points)


1. How many squares do you see in the logo? three

Describe where you see the squares.
1. Bottom left corner, has one triangle and a quadrilateral (BCIH)
2. Right-bottom to center of figure, has two triangles & a quad inside (GIEF)
3. Rotated square that runs through both squares. (ABDF)

2. The logo designer colored two triangles in the logo.

How are the two triangles related? the triangles are congruent
Justify your answer.

S	R
1. BCIH, BDFA, GIEF are sq.	1. Given
2. ∠BCI, ∠FED, ∠GFE, ∠FDB are rt ∠s	2. All angles in □ are rt
3. $\overline{BD} \cong \overline{DF}$	3. Sides are ≅ in □
4. $\overline{GF} \parallel \overline{IE}$	4. Opposite sides ∥ in square
5. ∠FDI ≅ ∠DFK	5. Alt. Int. ∠s ≅ when parallel lines
6. EF ⊥ GK	6. Def ⊥
7. ∠EFK is a rt ∠	7. Def ⊥
8. ∠EFK ≅ ∠BDF; ∠EFD ≅ ∠BCD	8. All rt ∠s ≅

9. ∠BDC ≅ ∠EFD | 9. Sub. prop
10. △BCD ≅ △DEF | 10. AAC

Performance Task The Company Logo P 1
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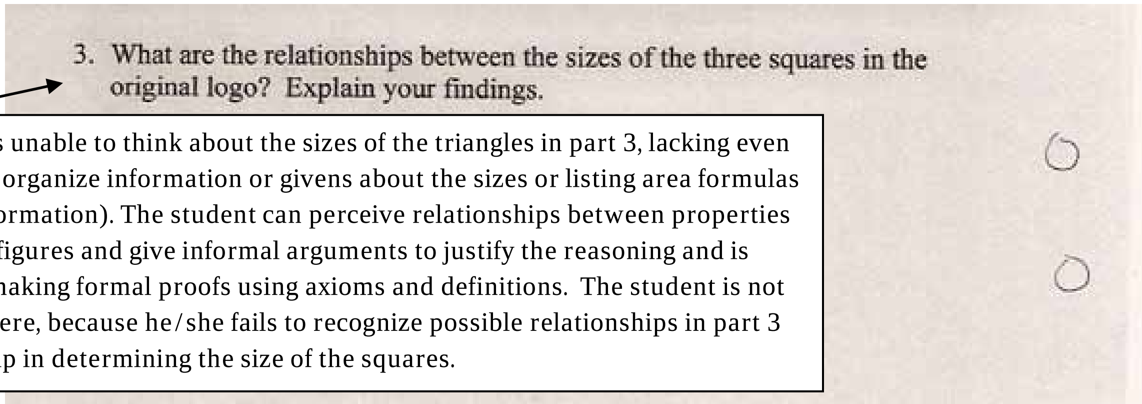
The student is able to identify the squares and locate them in space, using letters to help clarify the exact location of the squares, e.g. square BCIH. (diagram literacy)

In part 2 the student makes the conjecture that the triangles are congruent and develops the logic to support the conjecture using definitions and congruence theorems. The diagram is marked to show equal sides and angles as the proof is developed. (MP #3 – construct arguments, CCSS – congruence) The student successfully completes the core mathematics of the task.

Student B (cont'd)


3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

The student is unable to think about the sizes of the triangles in part 3, lacking even an attempt to organize information or givens about the sizes or listing area formulas (CCLS- transformation). The student can perceive relationships between properties and between figures and give informal arguments to justify the reasoning and is moving into making formal proofs using axioms and definitions. The student is not completely there, because he/she fails to recognize possible relationships in part 3 that might help in determining the size of the squares.

Implications for Instruction Level 3

Students at this level are starting to put together formal proofs and develop the logic of a proof. However they don't seem to be able to quantify geometric relationships about size. Students need to think about how to measure geometric shapes using area and volume. While the areas of these figures are all different, students who are good at composing and decomposing shapes, should be able to think about transformations to show that the area of the smaller squares sum to the area of the larger squares.

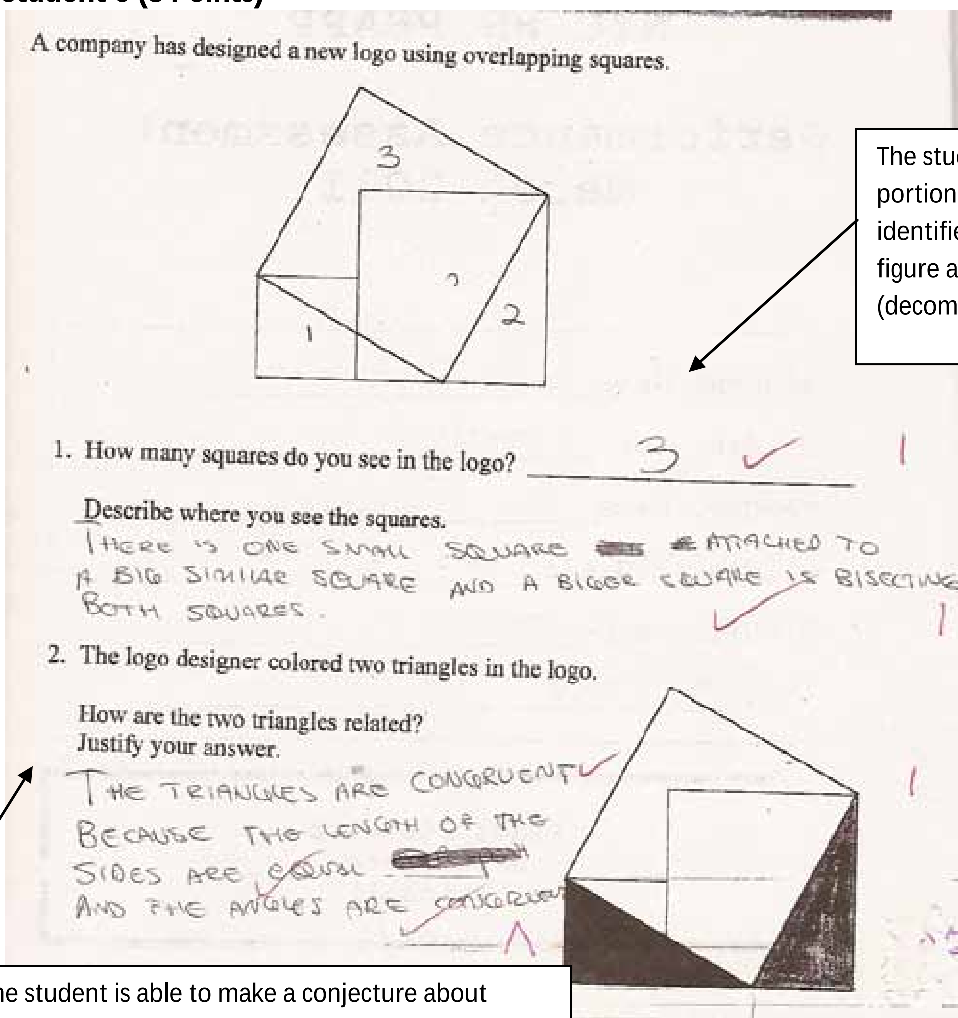
Students need more opportunities to apply knowledge of area and transformation to new situations and contexts. Asking students a simple question, "I overheard a student in another class say that part 3 could be proved using a transformation. What do you think she meant? How could you prove that this is true?" might be enough to get students at this level to push through to a solution. However they need more opportunity to work with open problems, so that they start to think through all the tools and procedures they know and pull out the appropriate strategies for themselves.

High School Geometry: Company Logo
Annotated Student Work

Level 2: Performance Below Standard – Score Range 3-4

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Student C (3 Points)


A company has designed a new logo using overlapping squares.

1. How many squares do you see in the logo? 3

Describe where you see the squares.
THERE IS ONE SMALL SQUARE ATTACHED TO A BIG SIMILAR SQUARE AND A BIGGER SQUARE IS BISECTING BOTH SQUARES.

2. The logo designer colored two triangles in the logo.

How are the two triangles related?
Justify your answer.
THE TRIANGLES ARE CONGRUENT BECAUSE THE LENGTH OF THE SIDES ARE EQUAL AND THE ANGLES ARE CONGRUENT

The student is able to do the entry-level portion of the task. In part 1, the student identifies the number of squares in the figure and where they are located (decomposing shapes).

In part 2 the student is able to make a conjecture about congruency (CCSS – congruence) and give general details about what would need to proved to justify congruency. The triangles would be congruent if all the sides were equal. If it were proved that the angles were equal that would help establish congruency. The student does not label sides or angles or give details that would justify these statements or the conjecture on congruency (MP #3 construct arguments, CCSS – congruence). The student is unable to move from general ideas about congruency to the specifics of these particular shapes.

Student C (cont'd)

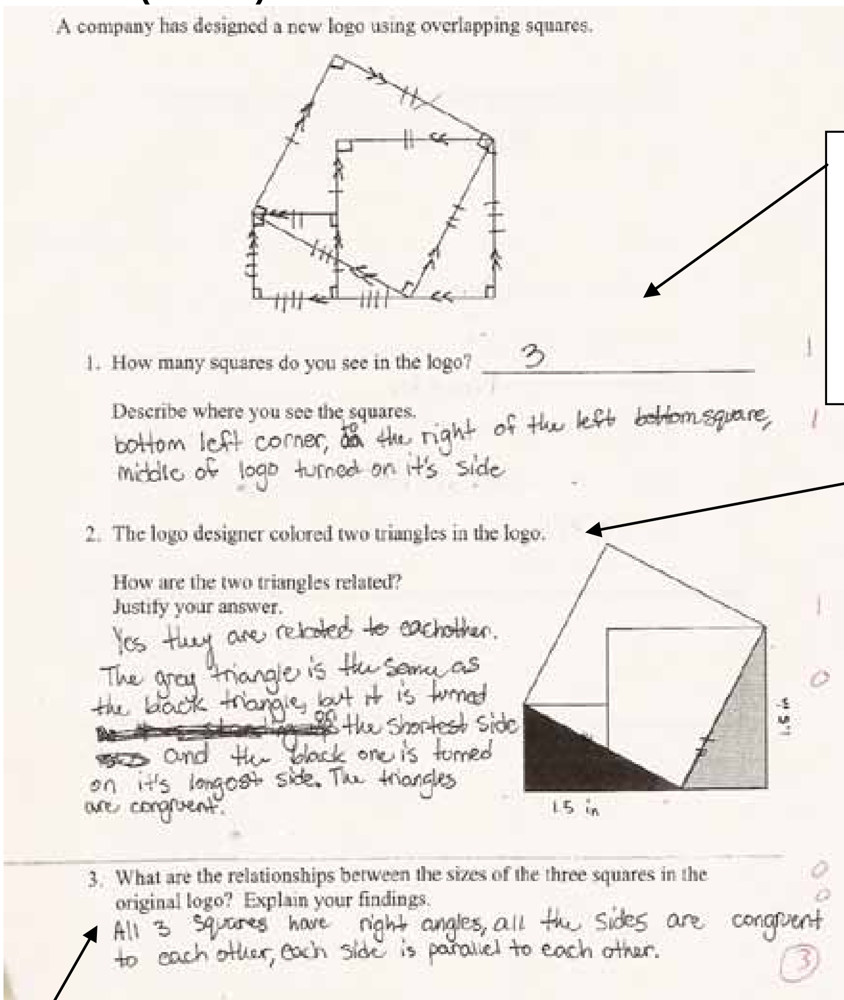
3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

THEY'RE ALL SIMILAR BUT NONE OF THEM ARE THE SAME SIZE. I HAVE LABELED THE SQUARES TO MAKE MY EXPLANATION CLEARER. SQUARE 2 IS DOUBLE THE SIZE OF SQUARE 1 AND ~~AND~~ SQUARE 3 IS 2.5 TIMES LARGER THAN SQUARE 3.

In part 3 the student makes an interesting conjecture about similarity, but then just estimates size relationships. The student seems to lack the critical mathematical perspective of using quantification, translation, calculation, or logic to justify sizes (MP#1- perseverance, MP#3 construct arguments, CCSS – similarity, congruence, transformation). The student can give some meaningful definitions, but can't put together a chain of reasoning to build an argument.

High School Geometry: Company Logo
Annotated Student Work

Student D (3 Points)



A company has designed a new logo using overlapping squares.

1. How many squares do you see in the logo? 3

Describe where you see the squares.
bottom left corner, ~~to~~ the right of the left ~~bottom~~ square, middle of logo turned on it's side

2. The logo designer colored two triangles in the logo.

How are the two triangles related?
Justify your answer.
Yes they are related to eachother. The gray triangle is the same as the black triangle, but it is turned ~~[illegible]~~ on the Shortest Side ~~[illegible]~~ and the black one is turned on it's longest side. The triangles are congruent.

1.5 in

1.5 in

3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.
All 3 squares have right angles, all the sides are congruent to each other, each side is parallel to each other.

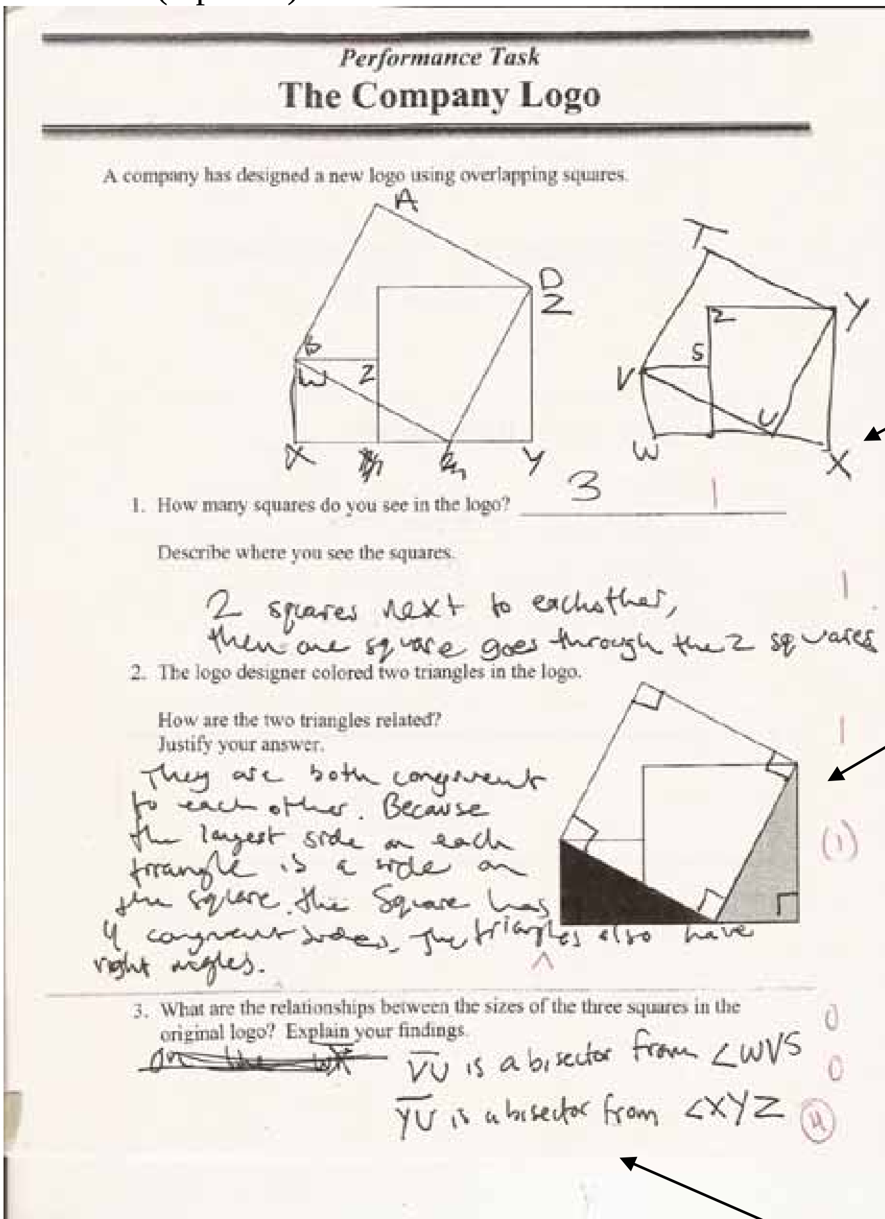
The student is able to do the entry-level portion of the task. In part 1, the student identifies the number of squares in the figure and explains where they are located in the diagram (decomposing shapes).

In part 2, the student starts by thinking about how the triangles could be related by using transformations. The student then makes the conjecture that the triangles are congruent. The student draws lines on the hypotenuses of the right triangles to indicate they are the same length, but the student does not make a clear statement of this fact or offer justification. The student attempts to use rotation and slides to show congruency. The student also uses measurement, 1.5 inches rather than deduction to show the longer legs are equal (MP#6 precision). As many diagrams are not drawn to scale and measurement can be imprecise this is not sufficient justification for this grade level (MP #3/ CCSS – congruence).

In part 3, the student only uses the definitions of squares to show how the squares are related. The student does not search for deeper types of relationships (MP#1 – perseverance). The prompt for question 3 specifically asks students to think about the size of the squares. (CCSS – transformation) The student can give a list of properties of geometric figures in part 2, but doesn't see how they might be used to compare the sizes.

High School Geometry: Company Logo
Annotated Student Work

Student E (4 points)



Performance Task

The Company Logo

A company has designed a new logo using overlapping squares.

1. How many squares do you see in the logo? 3

   Describe where you see the squares.

   2 squares next to eachother, then one square goes through the 2 squares

2. The logo designer colored two triangles in the logo.

   How are the two triangles related? Justify your answer.

   They are both congruent to each other. Because the largest side on each triangle is a side on the square. the Square has 4 congruent sides, the triangles also have right angles.

3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

   ~~On the left~~ $\overline{VU}$ is a bisector from $\angle WVS$

   $\overline{YU}$ is a bisector from $\angle XYZ$

In part 1 the student identifies the squares and locates them in space. The student labels the figures to assist with developing a justification in later parts of the task. (diagram literacy)

In part 2 the student is able to make the conjecture that the triangles are congruent. The student makes an attempt at developing a justification by showing the hypotenuses are equal (sides of a square) and that both triangles have right angles (no justification). While these are the beginning steps for proving congruency the argument is incomplete (MP #3). More detail is needed to finish the proof using hypotenuse angle theorem. (CCSS – congruence).

The student response in part three is irrelevant to the stem of the task, how are the sizes of the squares related. The student can perceive some relationships between properties and between figures but can't use them to build a complete argument (MP #3).

Implications for Instruction for Level 2

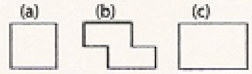
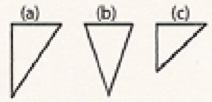
Students at this level should start to develop aspects of diagram literacy. Some students are able to think about side length and use the diagram to show equal lengths using hash marks. Students need to be pushed to think about using letters to identify vertices and talk about specific line segments to help them develop more complete justifications.

Students are noticing properties that might be important in developing a proof. They can make conjectures, such as, the triangles look congruent. However their justifications are not based in the specifics of the shapes in the diagram but are more closely related to a general definition of congruency. When they do give a step of the proof, the students don't have the habit of mind of continuing with "this is true because . . . " They don't back up their assertions.

Students at this level need frequent opportunities to make and test conjectures. They need to be asked questions, such as "Will this always work? Why or why not? Is this true for all triangles or just equilateral triangles?" Another activity for students at this level is to try and develop the smallest possible list of properties that will name or define only one type of shape. Having students work with warm ups, such as "always, sometimes or never true" is a useful strategy to help them develop informal logic. *(Note: A good resource for these types of activities is "Standards Unit: Improving Learning in Mathematics: challenges and strategies" which can be seen on-line by Googling the author Malcolm Swan. See example below.)*

(i) Odd one out

Perhaps the simplest form of classification activity is to examine a set of three objects and identify, in turn, why each one might be considered the 'odd one out'. For example, in the triplets below, how can you justify each of (a), (b), (c) as the odd one out? Each time, try to produce a new example to match the 'odd one out'.

(a) (b) (c) 	(a) sin 60° (b) cos 60° (c) tan 60°
(a) a fraction (b) a decimal (c) a percentage	(a) $y = x^2 - 6x + 8$ (b) $y = x^2 - 6x + 9$ (c) $y = x^2 - 6x + 10$
(a) (b) (c) 	(a) 20, 14, 8, 2, . . . (b) 3, 7, 11, 15, . . . (c) 4, 8, 16, 32, . . .

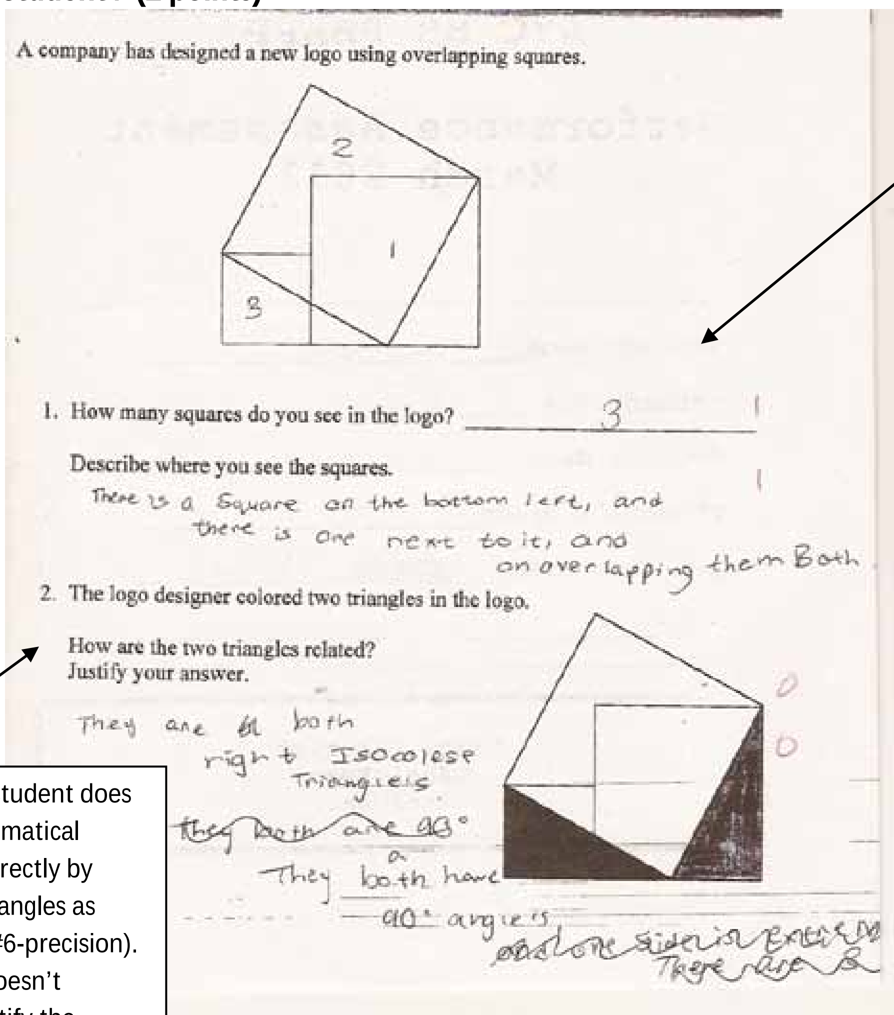
For example, in the first example, (a) may be considered the odd one as it has a different perimeter from the others, (b) may be considered the odd one because it is not a rectangle and (c) may be considered the odd one because it has a different area from the others.

High School Geometry: Company Logo
Annotated Student Work

Level 1: Demonstrates Minimal Success - Score Range 0- 2

The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The students struggle to recognize patterns or the structure of the problem situation.

Student F (2 points)


A company has designed a new logo using overlapping squares.

1. How many squares do you see in the logo? 3

Describe where you see the squares.

There is a Square on the bottom left, and there is one next to it, and on overlapping them Both

2. The logo designer colored two triangles in the logo.

How are the two triangles related?
Justify your answer.

They are both right Isocolese Triangleis

~~They both are 90°~~

They both have a 90° angle's

The student is able to complete the entry-level portion of the task. In part 1, the student identifies the number of squares in the figure and explains where they are located in the diagram (decomposing shapes).

In part 2, the student does not use mathematical definitions correctly by naming the triangles as isosceles (MP#6-precision). The student doesn't attempt to justify the assertion of 90° angles (MP#3-construct arguments). The student is not moving toward CCSS mastery by finding grade-level appropriate relationships and giving convincing arguments to back up conjectures in part 2 (CCSS Geometry – Congruence).

High School Geometry: Company Logo
Annotated Student Work

Student F (cont'd)

3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

~~They are all the same shape,~~
~~but different~~ 0

The 3 squares 0
in the
original logo,
have no relationship,
they are
Diffrent
Size's.

They have 2 equal side's.
~~Has 2 congruent legs~~
~~2 congruent~~ sides.

2

In part 3 the student notices that the squares are different sizes, but has no approaches to investigate, to calculate, or to compare the sizes. (MP#1- perseverance, CCSS – transformations) The student sees some properties of the figure, but makes decisions based on perception, not reasoning. The student doesn't seem to know which properties are important.

High School Geometry: Company Logo
Annotated Student Work

Implications for Instruction: Level 1

Students need more experience with mathematical definitions and being precise about language. Students would benefit from word walls and developing class definitions for key terms. These definitions should deepen over time as students have experiences requiring more sophisticated use of the terms. Students need to be familiar with types of triangles, either sorting by angles: acute, right, or obtuse triangles, or sorting by side length: scalene, isosceles, and equilateral. Students at this level are not using words, such as congruence, that are critical to the task.

Students at this level have difficulty identifying key properties of shapes and understanding which properties are critical to the problem at hand. They are content with giving a list of properties they see. Students need activities that help them identify properties of figures. Students need to think about not just individual shapes or figures but categories of shapes. Asking questions, such as "If a shape has four congruent sides will it always be a square? Why? Can you find a counterexample?", help to move the students to informal analysis. Students would also benefit from classifying and sorting activities where they get to look at properties of shapes and understand the underlying logic of the classifications. They need opportunities to think about how these shapes are alike and different, so they become more discerning in what properties they pay attention to.

Students seem to know that shapes are different sizes. They need to be asked how we can quantify or measure size of geometric shapes. They need to make the connection between area formulas as a tool to get an answer and as a tool for measuring shapes and comparing their sizes.

HIGH SCHOOL ALGEBRA: AUSSIE FIR TREE
INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline- High School Math

INTRODUCTION: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

High School Geometry: The Company Logo

UNIT TOPIC AND LENGTH:

- The unit uses an investigation of rigid motions and geometric theorems to teach students how to verify congruence of plane figures and use the implications of congruence to solve problems and create proofs about geometric relationships. Students will demonstrate mastery of the content by making sense of the Company Logo Performance Task and persevering in solving the task. Suggested unit length is 3-4 weeks.

COMMON CORE LEARNING STANDARDS:

- **G.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- **G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- **G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant.

Standards for Mathematical Practice:

- **MP. 1** Make sense of problems and persevere in solving them.
- **MP. 3** Construct viable arguments and critique the reasoning of others.
- **MP. 6** Attend to precision.

BigIdeas/Enduring Understandings:

- Mathematicians recognize congruence of plane figures and are able to prove congruence using geometric theorems.
- Congruence of plane figures can be verified through rigid motions.
- Congruence can be used to solve problems and prove geometric relationships.

Essential Questions:

- How can congruent figures help me to prove theorems and make my proofs convincing?
- To what extent does my understanding of rigid motions help me to verify congruence?
- How can proving theorems and solving problems involving congruence help me understand mathematical proofs and relationships?

Content:

Congruence

- of corresponding parts
- minimum requirements for congruence in triangles
- geometric relationships
- as defined by rigid motions

Rigid Motions

- plane figures
- transformations (translations & rotations)
- congruence

Proofs

- properties of plane figures
- triangle congruence proof
- deductive and inductive reasoning
- algebraic proof
- applications
- pythagorean theorem

Skills:

- **Identify** corresponding parts of figures
- **Explain** sufficient information for congruence in triangles
- **Express** geometric relationships using algebraic representation

- **Transform** plane figures using rigid motions
- **Identify** congruence through rigid motions

- **Calculate** the area of basic plane figures
- **Apply** the Pythagorean Theorem to identify right triangles and find their missing sides
- **Create** and **justify** valid arguments
- **Formulate** and **explain** various proofs of the Pythagorean Theorem
- **Recall** properties of plane figures to reason

KEY TERMS AND VOCABULARY:

- plane, proofs, theorem, congruence, rigid motion, Pythagorean Theorem, translations, rotations, transformations, corresponding parts

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT : RIGHT TRIANGLES

The ***initial assessment*** also allows for what is sometimes called a *touchstone task*. The task should be rich enough that it can be solved from a variety of approaches, so that students can make sense of it in natural ways. Then as the unit progresses, students should be able to move to more efficient or grade-level appropriate strategies. As the students learn new ideas or procedures, students and the teacher can reflect upon how these new ideas and procedures might apply to the initial task. See the task *Right Triangles* for task details.

FORMATIVE ASSESSMENT LESSON: PROOFS OF THE PYTHAGOREAN THEOREM

Use this **Formative Assessment Lesson** about 3/4 of the way through the unit. The purpose is to surface misconceptions and, through the course of the lesson, to provide ways for students to resolve these misconceptions and deepen their understanding. By surfacing misconceptions, the teacher is then able to make mid-unit corrections to instruction. Thus, students' experiences help to improve learning, rather than waiting until the final assessment to uncover problems or gaps in learning. See the formative assessment lesson *Proofs of the Pythagorean Theorem* for full details.

FINAL PERFORMANCE TASK: THE COMPANY LOGO

At the end of the unit, students will be given *The Company Logo* to determine how they have improved their thinking and mathematical skills over the course of the instructional unit. This task assesses students' ability to identify and locate figures in space, make and test conjectures about congruent figures and develop convincing arguments using valid geometric theorems, use area formulas and apply the Pythagorean Theorem, and construct proofs and understand the role of axioms and definitions. *See The Company Logo for full details.*

LEARNING PLAN & ACTIVITIES:

Triangles and Squares is the long task for this unit that should be embedded between the initial assessment (*Right Triangles*) and the Formative Assessment Lesson, *Proofs of the Pythagorean Theorem.* This long task allows students to work with area models to discover and examine relationships between geometric figures. Students use their understanding of these relationships, rigid motions, and/or algebraic representation to create a proof of the Pythagorean theorem. Patty paper and grid paper should be made available for this activity. See the task *Triangles and Squares*

for full details.

Additional Task: There is an additional task that can be used in conjunction with the formative Assessment Lesson *Proofs of the Pythagorean theorem*. The task bears the same name as the lesson, and should be given before the lesson is taught. This additional task examines three attempts to prove the Pythagorean theorem, and gives students an opportunity to analyze and evaluate the validity of different methods of proof and consider examples of effective modeling. See *Proofs of the Pythagorean Theorem Additional Task* for full details.

Think/Write/Pair/Share is a high level strategy that respects individual time to process and organize ideas before engaging in peer-to-peer discussions. This process can be used throughout the unit as a vehicle for students to self-reflect, construct new meaning by building on the ideas of others, and strengthen their arguments.

"Stop and Jots" and Journal Entries for Reflection: Using a prompt such as, "How has my thinking changed as a result of what I have discussed with my peers?" or "How can I improve my argument or explanation using evidence and appropriate vocabulary?" can provide valuable opportunities for students to improve their own solutions and deepen their understanding of the content.

Geometry Software is a powerful tool whether it is used by the teacher or put into the hands of the students. Experimentation with rigid motion is a highly effective application of such software and is encouraged within the unit. Some suggested computer programs include:

Cabri http://www.cabri.com/
Geometer's Sketchpad http://www.dynamicgeometry.com/
Cinderella http://www.cinderella.de/tiki-index.php

Restatements or Descriptions of proofs and methods of proof are extremely valuable in helping a student organize and contextualize new understandings. This can be applied during class discussion by asking students to restate the thinking of others in their own words, or students could be asked to describe the learning of the day or a specific method of proof that they have analyzed in class.

Purposeful Questioning and Feedback are instructional supports that help refocus students' attention on specific aspects of their work. The table on page 3 of the Formative Assessment Lesson *Proofs of the Pythagorean Theorem* provides some suggestions based on some common difficulties. Although these issues and questions related directly to the lesson, they can be easily modified to address similar misconceptions that may arise over the course of the unit.

RESOURCES:

Websites and Web-tools used

http://www.themathlab.com/geometry/mathcourt/writeproofs.htm
http://mathforum.org/dr.math/faq/faq.proof.html
http://www.buzzle.com/articles/history-of-pythagorean-theorem.html
http://www.buzzle.com/articles/pythagorean-theorem-formula.html

Materials Used

Right Triangles task
Triangles and Squares task
Proofs of the Pythagorean Theorem formative assessment lesson
Proofs of the Pythagorean Theorem additional task
The Company Logo task
Patty paper and Grid paper

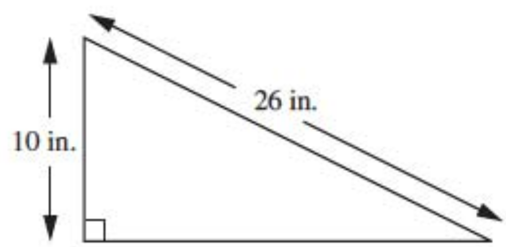
Right Triangles

This problem gives you the chance to:
- use the Pythagorean theorem to solve problems

Mario and Hank are arguing. Mario says it is impossible to draw a right triangle with sides measuring 8 inches, 12 inches, and 18 inches.
Hank says it is possible.

1. Who is correct? ____________________
Show your calculations.

2. Hank says it is possible to draw a right triangle with the measurements shown in the diagram below.



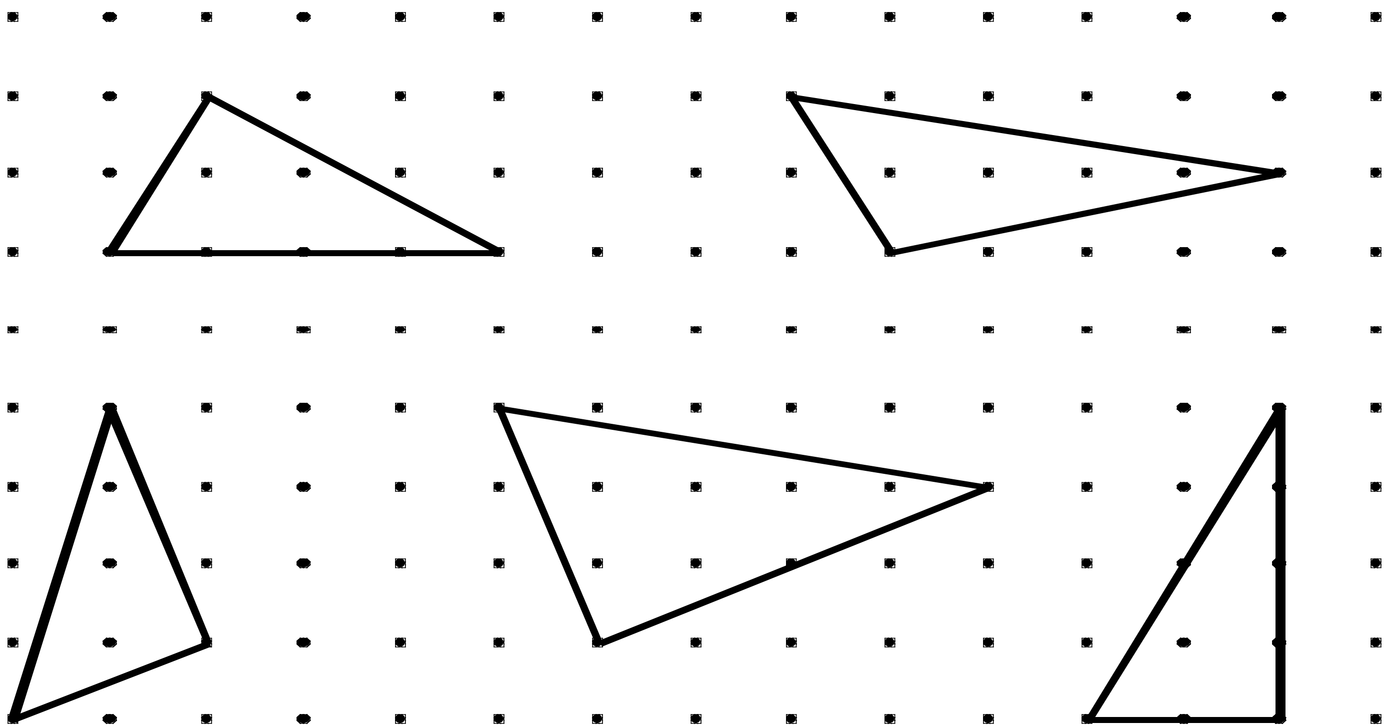
What is the length of the third side of this right triangle? ____________________
Show how you figured it out.

5

Triangles and Squares

Level C

Find the area of these triangles.

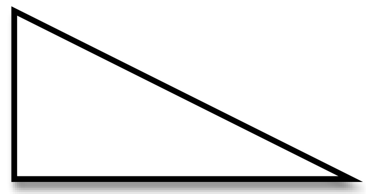


Explain your reasoning.

Triangles and Squares

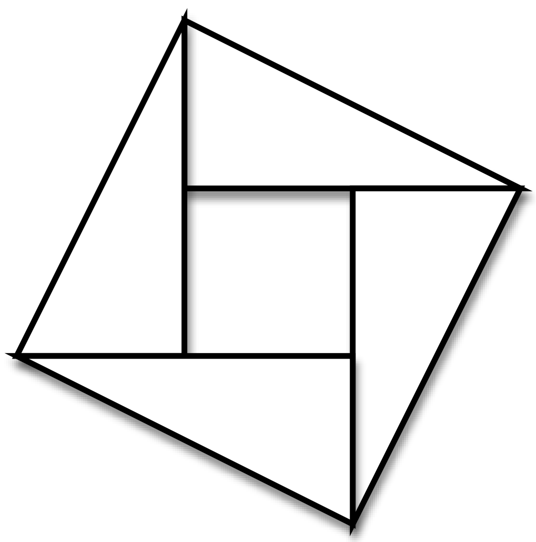
Level D

Triangle T is a right triangle.



Triangle T

The squares below are formed by four triangles that are all congruent to *Triangle T*.

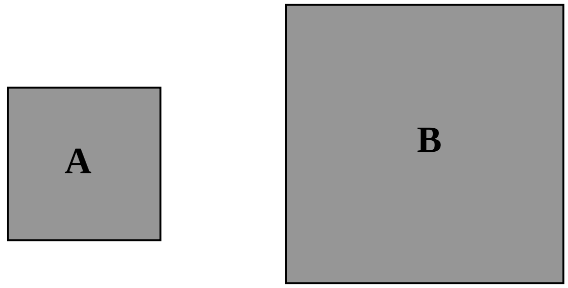


Use the squares and triangles above to prove the Pythagorean Theorem.

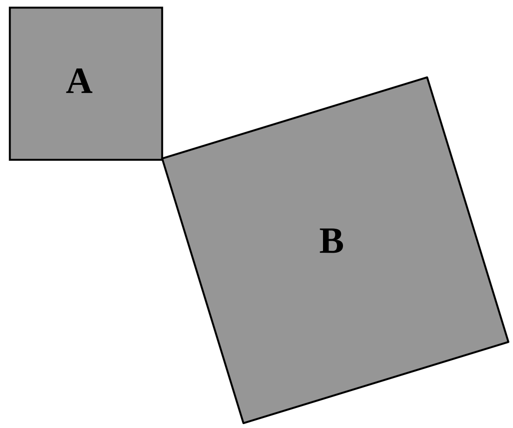
Triangles and Squares

Level E

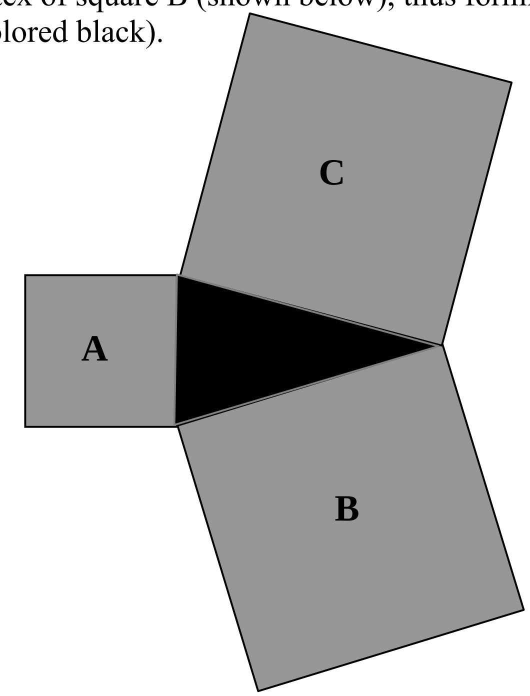
Here are two squares A and B



They can be arranged to share one vertex that forms an angle between 0º and 180º.

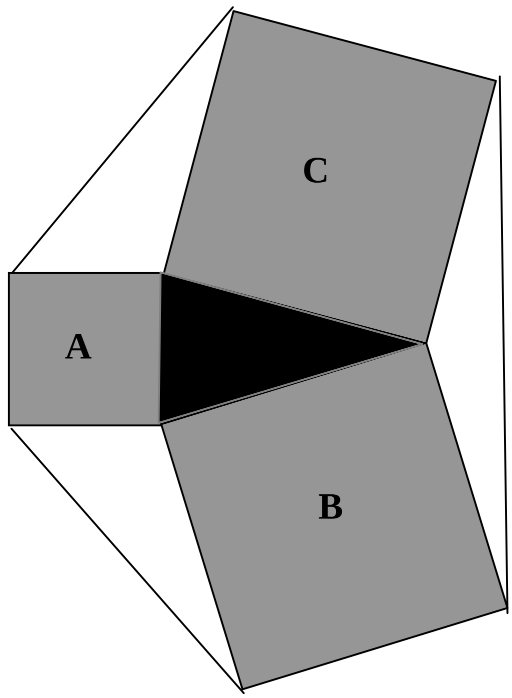


A third square C can be found which has a side length equal to the distance between a different vertex of square A to a different vertex of square B (shown below), thus forming a triangular region between the three squares (colored black).



Triangles and Squares

A line segment can be drawn between the vertices of the squares, forming three more triangles (each shaded white).



What are the relationships between the three white triangles? What is the relationship between the black triangle and the three white triangles? Explain and justify what you know.

Proofs of the Pythagorean Theorem

Mathematical goals

This lesson unit is intended to help you assess how well students are able to produce and evaluate geometrical proofs. In particular, this unit aims to identify and help students have difficulties in:

- Interpreting diagrams.
- Identifying mathematical knowledge relevant to an argument.
- Linking visual and algebraic representations.
- Producing and evaluating mathematical arguments.

Standards addressed

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

G-C: Prove geometric theorems.
10. Prove theorems about triangles.

This lesson also relates to the following *Mathematical Practices* in *the Common Core State Standards for Mathematics:*

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Introduction

The unit is structured in the following way:

- Before the lesson, students attempt the task individually. You review their work and formulate questions for students to answer, to help them improve their work.
- The lesson begins with a whole class discussion of a related diagram. Students then work collaboratively in pairs or threes on the assessment task, to produce a better collective solution than those they produced individually. Throughout their work they justify and explain their decisions to peers.
- In the same small groups, students critique examples of other students' work.
- In a whole class discussion, students explain and evaluate the arguments they have seen and used.
- Finally, students work alone to improve their individual solutions.

Materials required

- Each individual student will need a copy of the two task sheets, *Proofs of the Pythagorean Theorem (1)* and *(2)*. Provide a copy of the *Grid Paper* if the student asks for it.
- Each small group of students will need a new copy of the second task sheet, *Proofs of the Pythagorean Theorem (2)*, sheets of paper for working, and copies of the three sheets *Sample Responses to Discuss (1)*, *(2)*, and *(3)*. Provide copies of the *Grid paper* and extension task, *Another Proof of the Pythagorean Theorem*, as necessary.
- There are some projectable resources to help with whole-class discussion.

Time needed

Ten minutes before the lesson, and a one-hour lesson. Exact timing will depend on the needs of your students.

Before the lesson

Proofs of the Pythagorean Theorem task (10 minutes)

Set this task in class, or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess students' work and identify the forms of help they need. You will then be able to target your assistance more effectively in the formative assessment lesson.

Give out the task *Proofs of the Pythagorean Theorem (1) and (2)*.

Introduce the task briefly and help the class to understand the work they are being asked to do.

- *Spend ten minutes on your own working on these questions.*
- *You choose one diagram only to work with through the task.*
- *The first sheet has questions. Answer these on the second sheet.*
- *Write all your reasoning on the sheet, explaining what you are thinking.*
- *I have grid paper, if you want to use it.*

It is important that, as far as possible, students answer the questions without assistance.

Students who sit together often produce similar answers, then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. At the beginning of the formative assessment lesson, allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' written work for formative assessment. Read through their scripts and make informal notes on what their work reveals about their current levels of understanding and their different approaches to producing a proof.

We strongly suggest that you do not write grades or marks on students' work. Research shows that this is counterproductive, as it encourages students to compare grades and distracts their attention from what they are to do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work, or, if you do not have time for this, just select a few questions that will help the majority of students. These can then be written on the board at the end of the lesson.

Common issues:	**Suggested questions and prompts:**
Inaccurate construction of diagram For example: The student has joined vertices or sides in a different configuration. Or: The geometrical figures are not right triangles of equal size, or trapezoid has non-parallel sides.	• *Look carefully at the diagram Nadia drew. Which vertices are joined? Which sides?* • *Label the sides of the triangles in Nadia's diagram. Use this to help you draw accurately.*
Insufficient mathematical knowledge elicited For example: The student does not write the formula for the area of a square (Diagram 1), or the area of a trapezoid (Diagram 2).	• *Start by writing anything you know about squares/triangles/trapezoids.* • *Write things you know that link with this diagram.*
Use of relevant mathematical structure For example: The student has written that the area of a trapezoid is $\frac{h}{2}(a+b)$ but does not attempt to use this in the proof (Diagram 2). Or: The student does not recognize that the side length of the large square is the sum of a and b (Diagram 1).	• *How does your area formula apply to the trapezoid/square in your diagram?* • *Which lengths could you use to calculate the area of the large square? Why?* • *How could you write the length of this side using algebra?*
Incomplete solution For example: The student has written some relevant theorems and noticed some relevant structure but has lost direction.	• *What do you already know?* • *What do you want to find out?* • *Try working backwards: what will the end result be?*
Visual solution For example: The student rearranges the area c^2 to the configuration a^2+b^2.	• *Your rearrangement of the area shows $c^2=a^2+b^2$. Can you explain this in words and algebra, too?* • *Does your construction always work? How do you know with only this one diagram?*
Empirical solution For example: The student has measured the sides of the triangle and used those measures in length/area calculations.	• *Think of your triangle as representing <u>any</u> right triangle with sides a, b, c. How does your diagram help you show the Pythagorean Theorem is true for any right triangle?*
Incomplete reasoning For example: The student does not establish that the quadrilateral (Diagram 1) is a square of side length c, rather than a rhombus. Or: The student assumes the sides are parallel in the (supposed) trapezoid (Diagram 2).	• *Would someone in another class be convinced by your solution?* • *What reason do you have for thinking that this is a right angle? It is marked as a right angle, but how do you know for sure?*
Complete solution Provide copy of the extension task, *Another Proof of the Pythagorean Theorem.*	• *Here is a new diagram. Use it to prove the Pythagorean Theorem.*

Suggested lesson outline

Introduction (10 minutes)

Explain the structure of today's lesson. Remind students of their work on the Pythagorean Theorem in a previous lesson.

> *I read your solutions and I've some questions about your work on Proofs of the Pythagorean Theorem.*
> *You're going to use my questions to help you review your work at the end of today's lesson.*
> *First you're going to work together so you can answer my questions with confidence.*

Ask students to begin by stating and illustrating the Pythagorean Theorem. Work interactively with students, drawing diagrams on the board as necessary.

> *What is the Pythagorean Theorem?*
> *For what kind of triangles is it true? Is it any old triangle?*
> *What diagram could you draw to illustrate the Pythagorean Theorem?*
> *Are there any other ways of stating the theorem? [Prompt for both length and area expressions.]*
> *There are also many different ways of proving the Pythagorean Theorem.*

Show the students the diagram for *Bhaskara's construction* (projectable resource).

> *This is one diagram used for proving the Pythagorean Theorem.*
> *Explain how you'd construct it from one right triangle.*

Prompt students to use clear mathematical language to describe this construction process, such as *congruent, perpendicular, reflect in a horizontal/vertical line* and *rotate by 90° (counter) clockwise.* Research shows that students benefit from reconstructing diagrams for themselves.

Explain how students are to work together on this task.

- *In your groups, choose one of the diagrams and work together to produce a proof.*
- *It's the same task as you worked on alone, but together you can make a better solution.*

Explain that when proving, it is useful to gather knowledge of the mathematical structure:

- *First work out together how the diagram you choose is constructed.*
- *Then you need to think about what do you already know about squares, trapezoids, lengths, angles in general. Then think about what you know of this diagram in particular.*
- *Think about how you can use the diagram to figure out the statement of the Pythagorean Theorem.*
- *Make sure you explain everything really clearly.*

Collaborative group work (15 minutes)

Organize students into groups of two or three. Distribute a new copy of the task sheet to each group, with more paper for calculations and working.
You have two tasks during small group work:

- *Note different student approaches to the task.* Notice how students make a start on the task, if they get stuck, and how they respond if they do come to a halt. Note which theorems and properties of the square/right triangle/trapezium they identify, and how they choose which information may be useful. Notice when students look for different ways of writing the same information. Do they use visual/geometrical as well as algebraic language? Do they produce a general (algebraic or visual) solution, or do they introduce measures and work empirically?

- *Support student problem solving.* Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. If several students in the class are struggling with the same issue, write a relevant question on the board. You might also ask a student who has performed well on a particular part of the task to help a struggling student.

The following questions and prompts have been found most helpful:

- *Which side(s) of the triangle form(s) the side of the square?*
- *What is the length of this line segment?*
- *How could you calculate the area of this square? Is that the only way to figure out the area?*
- *How do you know this is always true? Maybe it only works for these numbers.*
- *You have assumed that…. Why do you believe that is true?*

If students have produced a proof from one diagram, ask them to work on the other proof. If a group decides it has adequate proofs based on both diagrams, provide the sheet, *Another Proof of the Pythagorean Theorem*.

Collaborative analysis of *Sample Responses to Discuss* (20 minutes)

Give each small group of students a copy of the *Sample Responses to Discuss*. These are two solutions produced students in trials.

- *Read through Penelope's and Marty's solutions. Neither of these is a perfect proof. Both have useful features.*
- *Figure out what you think the student is trying to do, and explain what each student could do to improve.*
- *Now evaluate the completed arguments. Which approach do you find most convincing? Why?*

This analysis task will give students the opportunity to engage with and evaluate different types of argument without providing a complete solution strategy. It also raises questions about what counts as a good mathematical argument for discussion in the plenary.

During small group work, support student thinking as before. Also, listen to see what students find difficult. Identify one or two aspects of these approaches to discuss in the plenary. Note similarities and differences between the sample approaches and those the students took in small group work. If students require more work, ask them to produce a proof using Penelope's diagram (*Garfield's construction*; projectable resource) with their improved version of Marty's (visual and algebraic) solution method.

Whole class discussion comparing different approaches (15 minutes)

Organize a whole class discussion to analyze the different methods of the *Sample Responses to Discuss*. The intention is that you focus on getting students to explain the methods of working, and compare different styles of argument, rather than checking numerical or algebraic solutions.

Choose one of the solutions as an initial focus for discussion. These are drawn separately in the projectable resources.

- *Penelope assumed the lines PQ and RS are parallel. How do you know she was correct?*
- *Marty assumed the angle in the inner quadrilateral was right. Was he correct? How do you know?*
- *(Eleni), your group used a similar solution method to Marty. Can you explain the solution for us?*
- *How could Marty/Penelope improve his/her solution?*
- *Was it hard to understand Marty's approach? Penelope's approach?*

Once students have explained both methods, ask them to compare them.

- *Which did you think was the most convincing proof? Why?*

Students improve their own work (10 minutes)

Invite students to revise their own work again. Hand students back their assessment tasks. If you have not written questions on individual scripts, display your list of questions on the board.

Ask students to use a different color of pen so that you and the student will be able to see what they have learned during the lesson.

If a student is satisfied with his or her solutions using both diagrams, ask the student to work on *Another Proof of the Pythagorean Theorem*, or on *Bhaskara's construction* (projectable resources).

Solutions

Proofs of the Pythagorean Theorem

Students may choose visual, empirical or algebraic approaches with any of the three diagrams. We provide sample algebraic proofs in the *Sample Responses to Discuss* (below).

Analysis of *Sample Responses to Discuss*

Marty's solution

Marty relies on visual transformation of the area. This visual transformation is a powerful mathematical tool, but it leaves too much for the reader to do; it does not constitute a full proof. Marty provides no explanation of his approach. He moves the four congruent triangles to form two rectangles of side lengths a and b, with two squares of sides length a, b respectively making up the rest of the large square area.

Marty could strengthen his solution by showing the connection between the variables he uses to label the side lengths and the areas of the constituent parts of the figures more explicitly. In particular, he needs to describe how the algebra he uses links the lengths a, b, c to the transformed areas in his second diagram. He needs to provide much more explanation of his work to make it clear for the reader.

For example, he could write:

"The side length of the large square is $a+b$. So the area of the large square is $(a+b)^2 = a^2+b^2+2ab$.

Now I can find the area of the individual pieces of the large square.

The inner square has side length c and area c^2.

Each of the right triangles has area $\frac{1}{2}ab$. Two of these triangles form a rectangle ab. There are four of them. So this gives an area of $2ab$ from the triangles.

Adding together all the pieces gives the area of the large square. So the area of the large square is c^2+2ab.

I now have two ways of writing the area of the large square. So $c^2+2ab= a^2+b^2+2ab$.

Subtract $2ab$ from each side to see that $c^2= a^2+b^2$."

Penelope's solution

Penelope gives a clear explanation of her approach at the beginning of her solution, that she is finding the area of the trapezoid as a whole, and as the sum of three triangle areas. She provides more information for the reader than Marty does, so her solution is a better *explanation*. However, she takes an empirical approach, using the measures of the sides of the triangles to find that the area is approximately equal whether found as a trapezoid or as the sum of the areas of three triangles. This is not a proof of the Pythagorean Theorem because it is not a general argument.

Penelope's solution can be improved using some of the formulas she identified.

However, she first needs to show that the quadrilateral is a trapezoid. The two (apparently) horizontal sides are parallel, since they meet the line segment ab at the same (right) angle.

Then area of the trapezoid = $\frac{1}{2}h(a+b) = \frac{1}{2}(a+b)(a+b) = \frac{1}{2}(a+b)^2 = \frac{1}{2}(a^2+2ab+b^2)$

Penelope can also find the area of the trapezoid as the sum of three triangles.

Area of triangle with side lengths a, b = $\frac{1}{2}ab$. There are two of these, with total area ab.

Penelope needs to establish that the angle between the segments of length c is right before using these to calculate the area of the triangle with side lengths c.

Since the triangle with sides *a*, *b* is right, the two unknown angles sum to 90°. The angle in the triangle with side lengths *c* forms a straight line with these two angles. So the missing angle in the triangle with side lengths *c* is 90°.

Area of triangle with perpendicular side lengths *c* is $\frac{1}{2}c^2$.

So the total area of the trapezoid is $\frac{1}{2}c^2 + ab$

Since both methods give the total area of the trapezoid, $\frac{1}{2}c^2 + ab = \frac{1}{2}(a^2 + 2ab + b^2) = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2$

Cancelling *ab* from both sides, $\frac{1}{2}c^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2$, so $c^2 = a^2 + b^2$ as required.

Another Proof of the Pythagorean Theorem

The extension task is another dissection proof. The sides *a*, *b* are perpendicular and so can be used to calculate the area of each right triangle. The hypotenuse of the right triangle, *c*, forms the side of the large square.

Area of large square = c^2.

Area of large square = area of four triangles + area of small square.

Area of one triangle = $\frac{1}{2} base \times \ perp.height = \frac{1}{2}ab$.

Area four triangles = $4 \times \frac{1}{2}ab = 2ab$.

Side length of inner square = $b - a$.

Area of inner square = $(b - a)^2 = b^2 - 2ab + a^2$.

So area of the large square is $c^2 = 2ab + b^2 - 2ab + a^2 = b^2 + a^2$ as required.

Proofs of the Pythagorean Theorem (1)

This page is a question sheet.

Record your solutions on the next page.

Nadia sketched these two diagrams.

Diagram 1

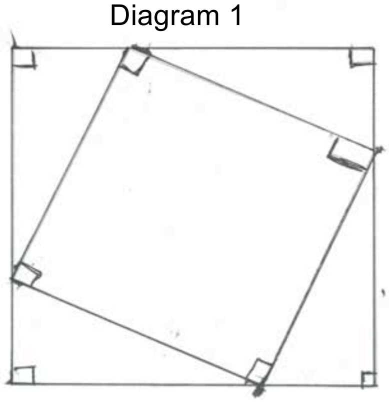
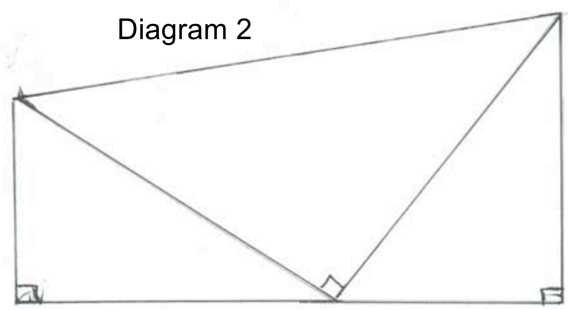


Diagram 2



1. Choose one of Nadia's diagrams.

On the grid on the next page there is a right triangle *abc.*

Use triangle *abc* to draw one of Nadia's diagrams accurately, on the grid.

2. Find information to help construct a proof of the Pythagorean Theorem:

(a) Write what you know about triangles, trapezoids and squares.

For example, you could write what you know about angles in a triangle, or how to calculate the area of a square or trapezoid.

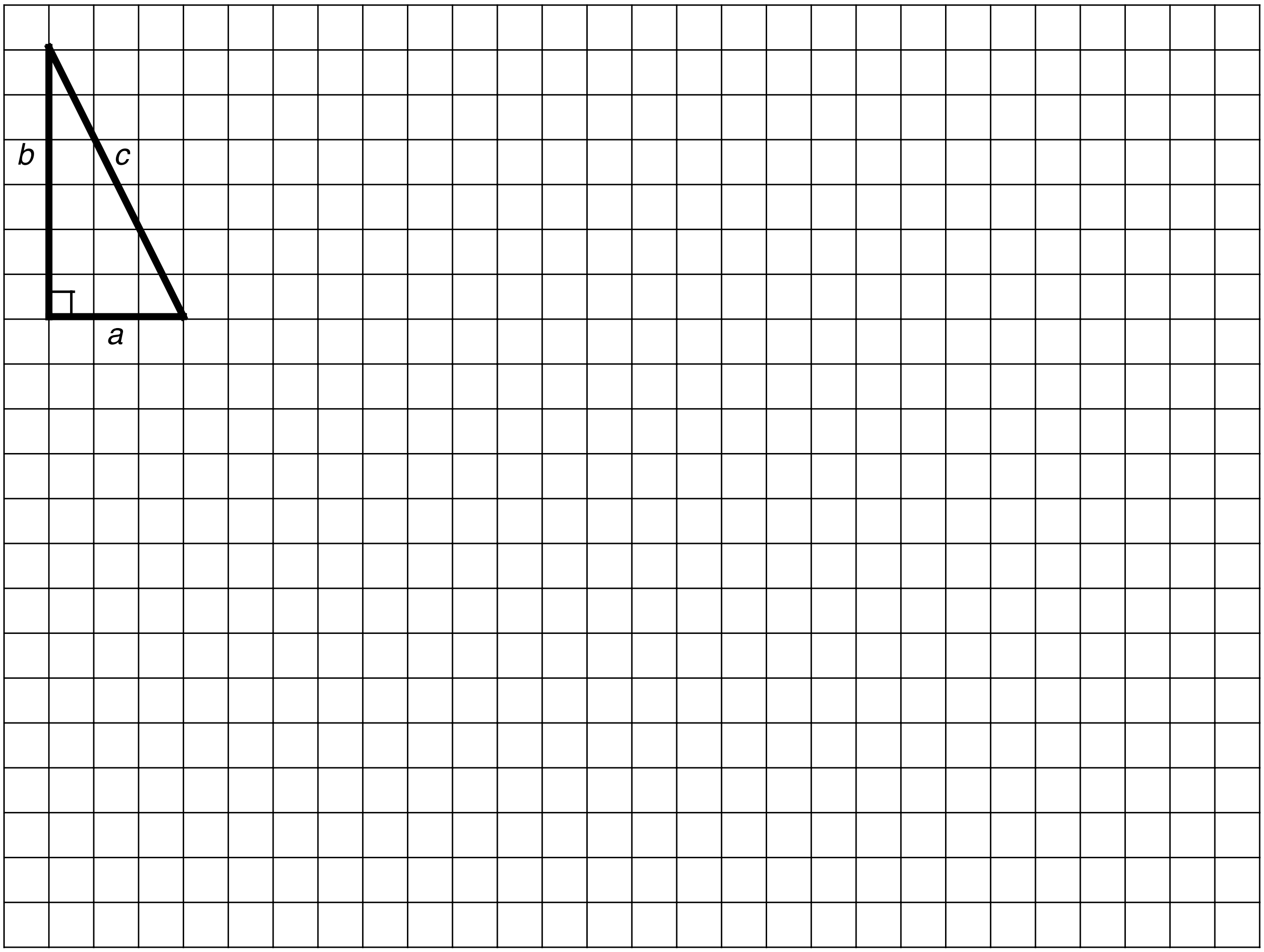
(b) Write what you know about your diagram.

For example, you could label lengths, find angles, or calculate areas.

3. Use your diagram to show that in any right triangle abc, $a^2 + b^2 = c^2$.

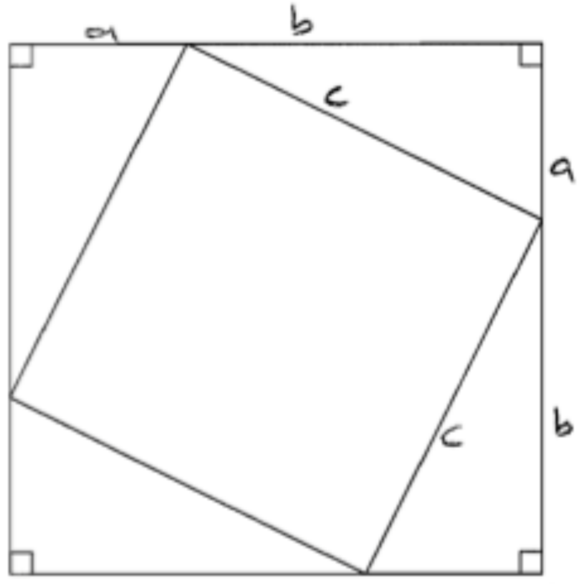
Proofs of the Pythagorean Theorem (2)

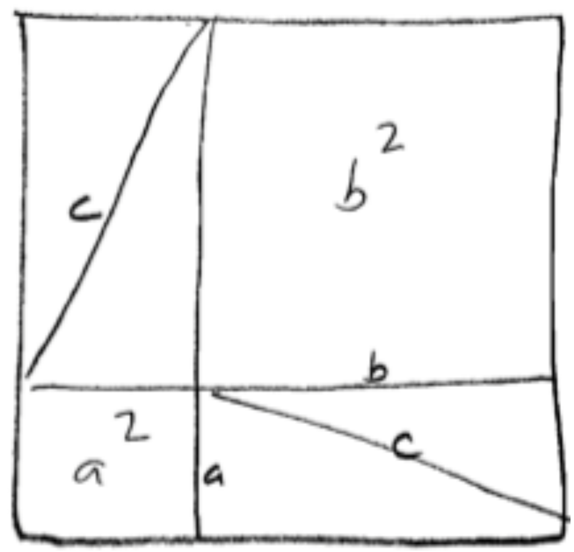
Write your solutions on this page.
Be sure to explain all your reasoning.
Your teacher has more grid paper if you need it.



Students Responses to Discuss (1)

Marty's solution





Describe Marty's solution method.

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Explain how he could improve his solution.

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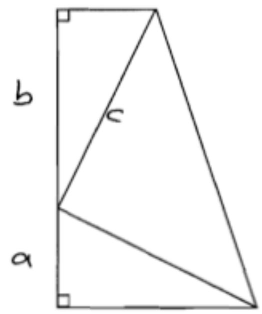
Students Responses to Discuss (2)

Penelope's solution

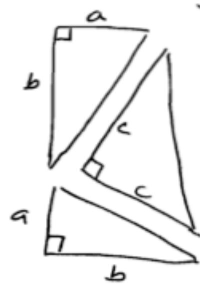
$a = 2cm$

$b = 4cm$

$c = 4.5cm$



This one trapezoid is three triangles!



Area trapezoid $= \frac{1}{2}$ height $\times$ (top + base)

$$= \frac{1}{2} \times (a+b) \times (a+b)$$

$$= \frac{1}{2}(a+b)^2$$

$$= \frac{1}{2}(2+4)^2 = \frac{1}{2} \times 36 = 18cm^2$$

Area triangle $= \frac{1}{2} \times$ base $\times$ perp. height

$$= \frac{1}{2} \times a \times b$$

$$= \frac{1}{2} \times 2 \times 4 = 4cm.$$

Two of them $= 8cm.$

Area other triangle $= \frac{1}{2} \times c^2 = \frac{1}{2} \times 4.5^2$

$$= 10.125cm^2.$$

$18.125cm^2 \approx 18 cm^2.$

Describe Penelope's solution method.

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Students Responses to Discuss (3)

Explain how Penelope could improve her solution.

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Compare Marty's solution with Penelope's solution.

Whose solution method do you find most convincing? Why?

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Produce a complete correct solution using your preferred method.

Your teacher has grid paper if you want to use it.

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Produce

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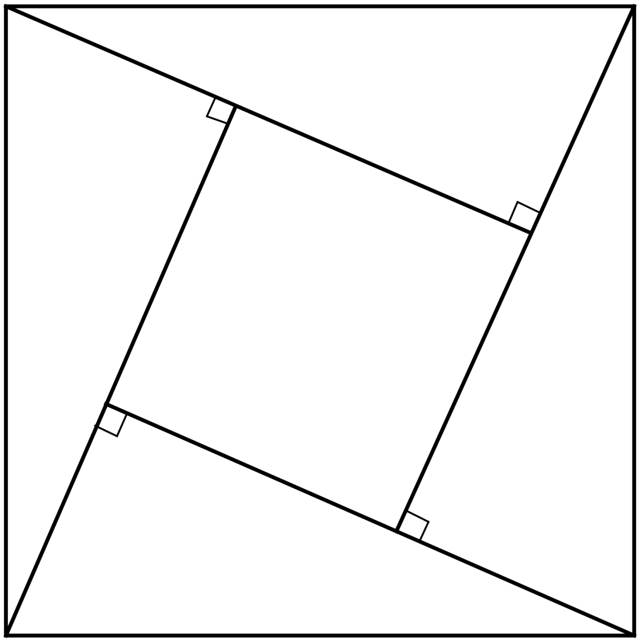
Grid Paper



Another Proof of the Pythagorean T

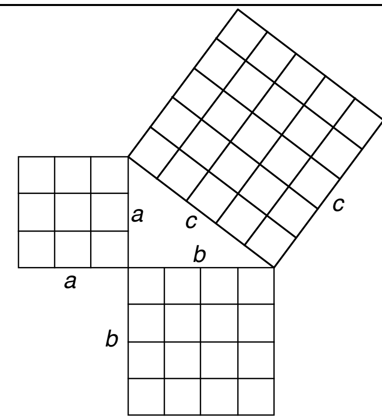
Proofs Of The Pythagorean Theorem?

Here are three attempts to prove the Pythagorean theorem.

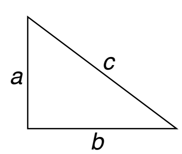
Look carefully at each attempt. Which is the best 'proof' ?

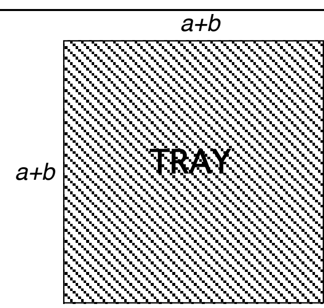
Explain your reasoning as fully as possible.

Attempt 1:
Suppose a right triangle has sides of length
a, b and *c*
Draw squares on the three sides as shown.
Divide these squares into smaller squares.
You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.
So: $a^2+b^2=c^2$

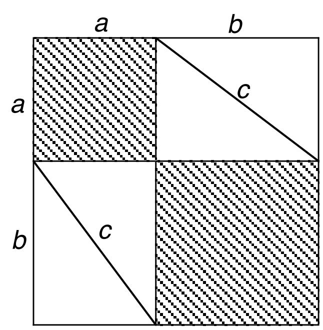


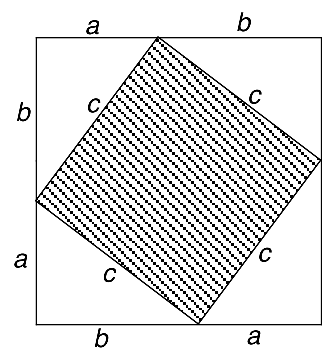
Attempt 2
Suppose that you start with **four** right triangles with sides of length *a, b* and *c* and a square tray with sides of length $a+b$.



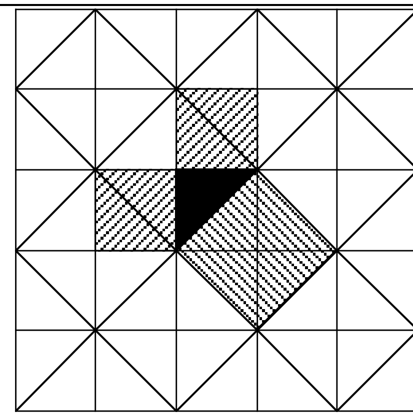


You can arrange the triangles into the tray in two different ways as shown here.
In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$.
In the second way you leave one large square hole. This has an area of c^2.
Since these areas are equal
$a^2 + b^2 = c^2$





Attempt 3:
The proof of the Pythagorean theorem is clear from this diagram.
The squares on the two shorter sides of the black triangle are each made from two congruent triangles.
These fit together to make the square on the longest side- the hypotenuse.



The best proof is attempt number ________

This is because

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My criticisms of the others are

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